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DETERMINING THE EFFICIENCY OF ATOMIZATION BY ITS FINENESS AND UNIFORMITY

By J. Sauter

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The mixtures of air and atomized fuel in internal combustion engines always contain drops of very different sizes. The efficiency of the fuel mixture is always increased:

1. By reducing the mean size of the drops, i.e., by increasing the fineness of the atomization;

2. By diminishing the differences between the sizes of the individual drops, i.e., by increasing the uniformity of the atomization. Great differences may cause disturbances in the flow and also in the combustion of the fuel mixture.

I. Determining the Fineness of a Mixture.

Since a mixture contains drops of very different sizes, the object is always to determine their mean size. This mean size can be defined in various ways, according to the object of its determination in the particular case (for example, the comparison of two mixtures with respect to their combustion characteristics).

* Die Beurteilung der Güte einer Zerstäubung nach ihrer Feinheit und Gleichmässigkeit." From "Forschungsarbeiten auf dem Gebiete des Ingenieurwesens," No. 279 (1926), pp. 4-15, published by the "Verein deutscher Ingenieure," Berlin. See also N.A.C.A. Technical Memorandum No. 390.

The mean size can be determined by two different methods. Either the sizes of a sufficient number of drops are measured and their mean calculated, or the mean size is found directly without knowing the sizes of the individual drops. The first method gives information on both the fineness and uniformity of a mixture, while the second method informs us only concerning the fineness of the mixture, by giving the mean size of the drops, without any indication concerning the uniformity of the mixture.

The distribution of the drops according to size can be represented graphically as follows. If we imagine as known the size and number (n) of the drops contained in a given volume of air and plot the different drop radii ρ as abscissas, we then obtain, by connecting the points thus located, the "frequency curve" of the drops (Fig. 1). For a mixture containing drops of only one size (radius ρ_0, number n), we thus obtain, instead of the curve, a point with the coordinates ρ_0 and n.

For example, it is assumed that the distribution represented by Fig. 1 was found in a mixture (perhaps by photography of a slowly moving jet). The drop radii therefore lie between $10\ \mu$ and $100\ \mu$, and the number of the drops decreases as their size increases. In order to obtain a criterion for the fineness of a mixture (e.g., of the one represented in Fig. 1), it is necessary to replace the drops of different sizes by drops of a definitely defined mean size. For such a definition there are six possibilities, which are subsequently given under Cases A to F. In order to facilitate the process, the following definitions are adopted in advance and it is thereby assumed that it is possible, for a given mixture, to determine the four quantities: number of drops, n; sum of the radii of the drops, L; total surface area of the drops, O; total volume of the drops, V.

By "mean size of the drops" is meant the resulting radius of the drops, when we imagine the tested mixture to be replaced by a perfectly uniform mixture with all the drops of one and the same size. Each pair of the four quantities (V, O, L, n) perfectly defines such a mean value and the remaining pair generally differs, for the substituted mixture, from the measured values in the tested mixture, since, for a mixture consisting of n_o drops of like radius r_o, we have:

$$V_o = r_o^3 \; n_o \; \frac{4}{3} \pi; \quad O_o = r_o^2 \; n_o \; 4 \pi; \quad L_o = r_o^1 \; n_o = r_o \; n_o; \quad n_o = r_o^0 \; n_o = n_o.$$

When two of the four values are known, then the value of r_o and n_o is always defined thereby and consequently also the values of the remaining two of the four quantities, so that they can no longer be chosen at will. Hence it is not generally possible to replace the tested mixture by a uniform mixture so that both mixtures will have the same values of V, O, L, and n, but the two mixtures will generally agree in only two of the above-mentioned values. We can accordingly replace the tested mixture by an equivalent mixture in various ways, according to our choice of the two values in which both mixtures are to agree. Since we can choose any two of the four values by six different methods, we can obtain six different values for the mean size of the drops, and it is therefore necessary to determine the importance of the individual values thus obtained.

Case A. L and n are the Same in Both Mixtures.

The most obvious method for finding the mean size of the drops is to take the arithmetical mean of their radii. Let $i_1, i_2, i_3 \ldots\ldots$ represent drops with radii $\rho_1, \rho_2, \rho_3 \ldots\ldots$ Then the mean value of the radius is

$$(r_m) = \frac{i_1\ \rho_1 + i_2\ \rho_2 + i_3\ \rho_3 + \ldots\ldots}{i_1 + i_2 + i_3 + \ldots\ldots};$$

If n denotes the total number of drops, then

$$(r_m) = \frac{i_1\ \rho_1 + i_2\ \rho_2 + i_3\ \rho_3 + \ldots\ldots}{n}.$$

$$(r_m) = \frac{\Sigma(i\ \rho)}{n} = \frac{L}{n}.$$

In order to obtain the value of (r_m), it is therefore necessary to determine the number n and the sum L of the radii of the drops.

The introduction of (r_m) replaces the actual mixture of n drops of various sizes by an imaginary mixture of n drops having the same radius (r_m). While, therefore, the number n and sum L agree in the two mixtures, the imaginary mixture has a different total volume and a different total surface area of the drops from those of the actual mixture. The mean value (r_m) thus obtained is therefore but poorly suited for the determination of the efficiency of the atomization for an engine, since not the sum of the radii, but the sum of the surface areas of the drops (n), formed from a certain volume V, is the criterion. Since the vaporization and combustion speeds depend on this and not on the sum of the radii, the mean value r_m affords no satisfactory basis for determining the vaporization characteristics of a mixture.

Case B. O and n are the Same in Both Mixtures.

Another mean value for the size of the drops is obtained by taking the arithmetical mean of the surface areas of the drops instead of the arithmetical mean of the radii. The former is $\frac{O}{n} = \frac{4\,\pi\,\Sigma\,(i\,\rho^2)}{n}$. The radius of the drops of an equivalent mixture, having n drops and a total surface area of O, is

$$(r_m)_1 = \sqrt{\frac{O}{4\,\pi\,n}} = \sqrt{\frac{\Sigma\,(i\,\rho^2)}{n}},$$

since $O = 4\,\pi\,n\,(r_m)_1^2$. The mean value $(r_m)_1$ accordingly represents the radius of the drops corresponding to the arithmetical mean of the surface areas.

Even this mean value is not suited to the determination of the vaporization characteristics of a mixture, since the mixture thus determined agrees with the given mixture only in the surface area and in the number of drops, but not in their volume nor in the sum of their radii.

Case C. V and n are the Same in Both Mixtures.

The same holds good for the mean value, when the arithmetical mean of the volume of the drops $\left(\frac{V}{n} = \frac{4\,\pi\,\Sigma\,(i\,\rho^3)}{3\,n}\right)$ is taken as the basis. The radius of the drops of a uniform mixture, having n drops with a total volume V, is

$$(r_m)_2 = \sqrt[3]{\frac{3\,V}{4\,\pi\,n}} = \sqrt[3]{\frac{\Sigma\,(i\,\rho^3)}{n}}, \quad \text{since}$$

$$V = \frac{4}{3}\,\pi\,n\,(r_m)_2^{\,3}.$$

The mean value $(r_m)_2$ accordingly represents the radius of the drops corresponding to the arithmetical mean of their volume. The mixture thus determined differs from the given mixture in the total surface area of the drops and therefore furnishes no criterion for the vaporization characteristics of a mixture, since the efficiency of a mixture, with respect to its combustion characteristics, increases with the total surface area of the drops obtained by the atomization of a given volume of liquid.

Case D. V and O are the Same in Both Mixtures.

Therefore it seems expedient to adopt, as the basis for determining the fineness of a mixture, a mean value for the size of the drops, which replaces the given mixture by an equivalent mixture which, with the same total volume, has the same total area of the drops as the given mixture. In this case we determine the actual total surface area of the n drops:

$$O = 4 \pi (i_1 \rho_1^2 + i_2 \rho_2^2 + i_3 \rho_3^2 + \ldots\ldots) = 4 \pi \Sigma (i \rho^2),$$

or their total cross-sectional area:

$$Q = \pi (i_1 \rho_1^2 + i_2 \rho_2^2 + i_3 \rho_3^2 + \ldots\ldots) = \pi \Sigma (i \rho^2),$$

and also the total volume V of the n drops, by determining, for example, the quantity of liquid atomized in producing them.

The mean value r_m thus obtained for the radius of the drops (whereby the actual mixture is replaced by an imaginary mixture which, with the same volume of the drops, has the same total surface area of the drops as the other) furnishes a correct criterion for the efficiency of the mixture. Therefore,

$$V = \frac{4}{3} \pi \Sigma (i \rho^3) \quad \text{and} \quad O = 4 \pi \Sigma (i \rho^2) \quad \text{or} \quad Q = \pi \Sigma (i \rho^2).$$

For the imaginary mixture, which is supposed to consist of n' drops of uniform radius r_m and to have the same value for the surface O and the volume V of the drops as for the actual mixture,

$$O = 4 \pi r_m^2 n' \quad \text{and} \quad V = \frac{4}{3} \pi r_m^3 n' \quad \text{and hence} \quad n' = \frac{3 V}{4 \pi r_m^3}.$$

Thereby n' is the number of drops in the imaginary mixture, which differs from the number n of the drops in the actual mixture, because the mean size of the drops, as already mentioned, has already been determined by two of the four quantities V, O, L, n, which define the mixture. Likewise the two mixtures differ with respect to the sum L of the radii. By substituting the value of n' we obtain

$$O = \frac{4\ \pi\ r_m^2\ 3\ V}{4\ \pi\ r_m^3} = \frac{3\ V}{r_m}$$

and

$$r_m = \frac{3\ V}{O} = \frac{\Sigma\,(i\,\rho^3)}{\Sigma\,(i\,\rho^2)} .$$

For the above assumed mixture, we have

Table I.

ρ in μ	i	i ρ	i ρ^2	i ρ^3
100	8	800	8×10^4	80×10^5
90	11	990	8.91×10^4	80.2×10^5
80	15	1200	9.6×10^4	76.8×10^5
70	19	1330	9.31×10^4	65.1×10^5
60	24	1440	8.64×10^4	51.8×10^5
50	31	1550	7.75×10^4	38.7×10^5
40	40	1600	6.4×10^4	25.6×10^5
30	53	1590	4.77×10^4	14.3×10^5
20	73	1460	2.92×10^4	5.84×10^5
10	100	1000	1×10^4	1.0×10^5
	374 = n	12960 $= \Sigma(i\,\rho)$	$67.3 \times 10^4 = \Sigma\,(i\,\rho^2)$	$439.34 \times 10^5 = \Sigma\,(i\,\rho^3)$

Hence:

$\Sigma\ (i)=374;\quad \Sigma(i\rho)=1.3\times10^4;\quad \Sigma(i\rho^2)=6.73\times10^5;\quad \Sigma(i\rho^3)=4.39\times10^7;$

$m=374;\qquad L=1.3\times10^4;\qquad O=8.46\times10^6;\qquad V=1.84\times10^8;$

and therefore:

$$(r_m) = \frac{\Sigma(i\rho)}{n} = \frac{1.3\times10^4}{374} = 34.7\ \mu$$

$$(r_m)_1 = \sqrt{\frac{\Sigma(i\rho^2)}{n}} = \sqrt{\frac{6.73\times10^5}{374}} = 42.4\ \mu$$

$$(r_m)_2 = \sqrt[3]{\frac{\Sigma(i\rho^3)}{n}} = \sqrt[3]{\frac{4.39\times10^7}{374}} = 49.0\ \mu$$

$$r_m = \frac{\Sigma(i\rho^3)}{\Sigma(i\rho^2)} = \frac{4.39\times10^7}{6.73\times10^5} = 65.4\ \mu$$

The difference between the different mean values is therefore quite large and the quantity r_m is almost twice as great as the quantity (r_m) corresponding to the arithmetical mean of the radii.

The small drops are far more numerous and the quantity (r_m) is therefore small. The large drops preponderate, however, in volume and surface area, as shown by Table I and Fig. 2. In the latter the values in Table I are plotted against the radius ρ, and the quantities i, $i\rho$, $i\rho^2$, and $i\rho^3$ are respectively represented by the curves I-IV. For the sake of clearness, the sizes of the different drops are represented by circles of corresponding magnitude.

From Fig. 2 it is obvious that, even with a small excess in the number of the small drops and hence for small values of (r_m), the atomization may be insufficient for internal combustion engines.

The mean value r_m can be reduced to the values $(r_m)_1$ and $(r_m)_2$. It is

$$r_m = \frac{\Sigma(i\rho^3)}{\Sigma(i\rho^2)} = \frac{\frac{\Sigma(i\rho^3)}{n}}{\frac{\Sigma(i\rho^2)}{n}} = \frac{(r_m)_2{}^3}{(r_m)_1{}^2} .$$

The value r_m therefore represents the quotient of the arithmetical mean of the volumes of the drops divided by the arithmetical mean of their surface areas. In addition to the above-mentioned quotient of two arithmetical means, two more can be developed in like manner, as will be shown in Cases E and F.

Case E. V and L are the Same in Both Mixtures.

If the uniform imaginary mixture and the given mixture agree in the values V and L, we then have, for the size of the drops, the mean value

$$(r_m)_3 = \sqrt{\frac{3\ V}{4\ \pi\ L}} = \sqrt{\frac{\Sigma(i\rho)^3}{\Sigma(i\rho)}} .$$

This expression is simplified as follows. The radius of the drops of the imaginary mixture is $(r_m)_3$. The number of the drops obtained from the volume V and having the radius $(r_m)_3$ is n_3. We then have

$$L = n_3\,(r_m)_3\,; \qquad V = \frac{4}{3}\,\pi\, n_3\,(r_m)_3^{\,3}\,, \quad \text{also} \quad n_3 = \frac{3\,V}{4\,\pi\,(r_m)_3^{\,3}}\,.$$

Hence:

$$L = \frac{3\,V}{4\,\pi\,(r_m)_3^{\,3}}\,(r_m)_3 = \frac{3\,V}{4\,\pi\,(r_m)_3^{\,2}} \quad \text{and}$$

$$(r_m)_3 = \sqrt{\frac{3V}{4\,\pi\,L}} = \sqrt{\frac{\Sigma(i\,\rho^3)}{\Sigma(i\rho)}} = \sqrt{\frac{\dfrac{\Sigma(i\,\rho^3)}{n}}{\dfrac{\Sigma(i\rho)}{n}}} = \sqrt{\frac{(r_m)_2^{\,3}}{(r_m)}}\,.$$

Case F. O and L are the Same in Both Mixtures.

If the uniform imaginary mixture and the given mixture agree in the values O and L, we then obtain, for the size of the drops, the mean value

$$(r_m)_4 = \frac{O}{4\,\pi\,L} = \frac{\Sigma(i\,\rho^2)}{\Sigma(i\,\rho)}\,.$$

If the imaginary mixture contains n_4 drops with a diameter of $(r_m)_4$, then

$$O = 4\,\pi\,n_4\,(r_m)_4^{\,2} \quad \text{and} \quad L = n_4\,(r_m)_4\,, \quad n_4 = \frac{L}{(r_m)_4}\,.$$

By substituting this value, we obtain:

$$(r_m)_4 = \frac{O}{4\,\pi\,L} = \frac{\Sigma(i\,\rho^2)}{\Sigma(i\rho)} = \frac{\dfrac{\Sigma(i\,\rho^2)}{n}}{\dfrac{\Sigma(i\rho)}{n}} = \frac{(r_m)_1^{\,2}}{r_m}\,.$$

For convenient comparison, the mean sizes of the drops corresponding to each pair of the four quantities V, O, L, n are given in Table II.*

Table II.

Given	Mean Value	Values for the example	Case
L and n	$(r_m) = \frac{\Sigma(i\rho)}{n} = \frac{L}{n}$	$(r_m) = 34.7\ \mu$	A
O and n	$(r_m)_1 = \sqrt{\frac{\Sigma(i\rho^2)}{n}} = \sqrt{\frac{O}{4\pi n}}$	$(r_m)_1 = 42.4\ \mu$	B
V and n	$(r_m)_2 = \sqrt[3]{\frac{\Sigma(i\rho^3)}{n}} = \sqrt[3]{\frac{3\ V}{4\pi n}}$	$(r_m)_2 = 49.0\ \mu$	C
V and L	$(r_m)_3 = \sqrt{\frac{\Sigma(i\rho^3)}{(i\rho)}} = \sqrt{\frac{3\ V}{4\pi L}}$	$(r_m)_3 = 58.2\ \mu$	E
O and L	$(r_m)_4 = \frac{\Sigma(i\rho^2)}{\Sigma(i\rho)} = \frac{O}{4\pi L}$	$(r_m)_4 = 51.8\ \mu$	F
V and O	$(r_m) = \frac{\Sigma(i\rho^3)}{\Sigma(i\rho^2)} = \frac{3\ V}{O}$	$r_m = 65.4\ \mu$	D

Regarding the importance of the determination of the mean sizes of the drops, it may be said in brief that, for determining the fineness of a mixture with respect to its efficiency as engine fuel, we are concerned only with the value r_m obtained from the determination of the volume of liquid atomized and the

*A more widely differing mean value is given on p. 18 of this number (279) of "Forschungsarbeiten," which is obtained from the dynamic pressure exerted by the drops. Since this mean value differs for the same mixture according to the location of the measuring point, it is omitted in the present article.

total surface area of the resulting drops. Its determination is, in fact, possible, as will be shown later.

The fineness of a mixture is determined by the value of r_m, i.e., by determining V and O, which shows nothing, however, regarding its uniformity. When, however, in addition to V and O, another one of the four quantities V, O, L, n is known, it is then possible to determine whether a given mixture is perfectly (or approximately) uniform. When three of the four quantities are known, then three of the six mean values are always known. (One of them can always be expressed by the other two.) It can be demonstrated that, only when a mixture is perfectly uniform, the individual mean values are of equal magnitude. When, for example, the quantities V, O, L are known, the mean values r_m and $(r_m)_3$ are also known

$$\left(\text{and likewise } (r_m)_4 = \frac{(r_m)_3^2}{r_m}\right).$$

If $r_m = (r_m)_3$, then the given mixture must be perfectly uniform. (The proof of this is given in the appendix on p. 72 of this number (279) of "Forschungsarbeiten.") We can therefore conclude, from the ratio of the two mean values that, when the quotient of the two mean values is 1, the mixture is perfectly uniform, while its value in nonhomogeneous mixtures always differs from 1.

In order to obtain a definite dimension, the concept of the degree of uniformity (or lack of uniformity) must be more accurately defined.

II. Determining the Uniformity of a Mixture

Any mixture which contains only drops of like size, is termed uniform, as heretofore. A mixture is considered as more lacking in uniformity, the more of its drops differ from a certain definite mean value R and the greater this difference is. The criterion for the deviations from the mean value R is designated as the degree of lack of uniformity.

In determining the degree of lack of uniformity (U-G), the given mixture is compared with a standard uniform mixture. The latter is obtained by letting the size of its drops represent a certain definite mean value R of the size of the drops of the given mixture. The manner of defining this mean value R depends on the object of the determination of the U-G. The U-G of a mixture can be determined with respect to the lack of uniformity of the radii of the drops, their surface areas or their volumes. In defining the U-G, we must, moreover, bear in mind that it must be independent of the unit employed in measuring the drops and must therefore be a nondimensional coefficient.

The U-G of the whole mixture is equal to the arithmetical mean of the values obtained for the individual drops. Its magnitude depends, for each drop, on its difference in magnitude in comparison with the adopted mean value R. Its magnitude can, indeed, be put proportional to the difference in size or, in general, proportional to the mth power of the difference in size. Thereby it does not concern the absolute difference in size, but only the percentile difference as compared with the adopted mean value R.

Two drops, one of which is greater by a certain amount than the mean value R, and the other smaller by the same amount, would give the same absolute contribution to the value of U-G. In a uniform mixture the value of U-G must be zero. Moreover, every contribution of a drop differing from zero must increase the U-G. The absolute values of the contributions of the individual drops are therefore to be added without regard to their signs.

The above considerations lead to the following definition of the U-G. Let b represent the contribution of one drop. Hence, $i\,b = B$ represents the contribution of i drops of the same size. Then U-G equals the arithmetical mean of the sum of the absolute contributions B, or

$$\frac{\Sigma\,[(\text{abs})\ B]}{n} = \frac{\Sigma\,[(\text{abs})\ i\ b]}{n},$$

in which n is the number of drops in the mixture under investigation.

The contribution of a drop of the size ρ to the U-G is proportional to the mth power of the percentile difference between R and ρ and hence proportional to the mth power of $\frac{R-\rho}{R}$. Thereby m is a positive whole number. This applies to the U-G with respect to the radii of the drops. For the U-G with respect to the surface areas of the drops, the contribution is the mth power of $\frac{R^2 - \rho^2}{R^2}$, hence $b = \left(\frac{R^2 - \rho^2}{R^2}\right)^m$. Correspondingly, the individual contribution with respect to the volumes is $b = \left(\frac{R^3 - \rho^3}{R^3}\right)^m$. In general, therefore, the contribution of a drop is $b = \left(\frac{R^a - \rho^a}{R^a}\right)^m$, in which we put $a = 1, 2,$ or 3, according to whether the determination of the U-G concerns the radii, surface areas or volumes of the drops. The U-G is therefore

$$\Phi = \frac{\Sigma[(abs)i\ b]}{n} = \frac{\Sigma\left[(abs)i\left(\frac{R^a - \rho^a}{R^a}\right)^m\right]}{n} = \frac{1}{n\ R^{a\,m}}\ \Sigma[(abs)i(R^a - \rho^a)^m].$$

The choice of the quantities a, m, and R depends on the object of the determination of the U-G.

The investigation of fuel mixtures for internal combustion engines involves the determination of the U-G with respect to the surface areas of the drops (hence $a = 2$). The quantity R can be chosen at will. It gives the size of the drops of the uniform mixture with which the given mixture is to be compared with respect to its U-G. We can adopt the arithmetical mean of the sizes of the drops or any other value, according to the object of the investigation. If, however, the fineness and uniformity of a mixture are to be determined, the same mean value must be taken as the basis, in order to obtain corresponding values, and hence (according to the preceding) for the determination of the excellence of mixtures with respect to their combustion characteristics, the mean value $r_m = \frac{3\ V}{O} = \frac{\Sigma(i\rho^3)}{\Sigma(i\rho^2)}$. Consequently, we put $R = r_m$ for the object of the present investigation.

The exponent m can be given any desired value. The greater it is, the greater effect each drop, with increasing difference in magnitude $R^a - \rho^a$, has on the U-G. For $m = 2$ we have especially simple relations, since the individual contributions of the drops to the U-G (as the squares of real numbers) are then always positive. We will therefore put $m = 2$ in what follows. With $a = 2$, $m = 2$, and $R = r_m$, we have

$$\Phi = \frac{1}{n\ r_m^4}\ \Sigma[i\ (r_m^2 - \rho^2)^2] = \frac{1}{n\ r_m^4}\ \Sigma[i\ (r_m^4 - 2\ r_m^2\ \rho^2 + \rho^4)],$$

and, since $\Sigma(i) = n$, we obtain

$$\Phi = 1 - \frac{2}{n\ r_m^2}\ \Sigma(i\rho^2) + \frac{1}{n\ r_m^4}\ \Sigma(i\rho^4).$$

This expression therefore represents the U-G of a mixture with respect to the surface areas of the drops. When the size of the individual drops of a mixture is given, the U-G Φ can be determined, but not when the values V, O, L and n are alone given, since the sum $\Sigma(i\rho^4)$ can not be expressed in these quantities. The determination of the U-G Φ can not therefore be reduced to the determination of the quantities V, O, L, n, or of the resulting six mean values, but must be calculated from the sizes of the individual drops.

We will now consider whether the above-defined U-G constitutes a suitable criterion for the determination of the uniformity of a mixture. We will then show that the definition of the U-G, although correct in itself, needs to be extended in one direction, in order to be available for the determination of the uniformity of mixtures.

In the deduction of the U-G, it was assumed in advance that two drops, one of which is greater by a certain amount than the mean value R and the other smaller by the same amount, would give the same contribution to the value of U-G. This assumption, however, is not the only possible one. It is only a special case of the more general form, in which a certain weight is ascribed to the individual contributions which make up the U-G, in that each contribution is multiplied by $(\rho/R)^k$. The previous U-G holds good for $k = 0$. If we put $k = 1$, we then obtain a U-G, in which the contribution of a drop depends both on the difference $R^a - \rho^a$ and also on ρ/R, the ratio of the size of the drop to the mean size R, so that the contribution to the U-G, of a drop which is small in relation to R, is correspondingly smaller than that of a drop which is large in relation to R. The same principle holds true for $k = 2$ and $k = 3$.

It is necessary to expand the definition of the U-G in this manner, for the combustion characteristics of a mixture are poorer, with the same U-G, when the lack of uniformity is due to large drops (as compared with the mean value R) than when it is due to small drops.

The contribution of a drop to the U-G is now

$$b = \left(\frac{R^a - \rho^a}{R^a}\right)^m \left(\frac{\rho}{R}\right)^k ,$$

and we accordingly obtain the U-G:

$$\Phi = \frac{\Sigma[(abs)i\ b]}{n} = \frac{1}{n}\Sigma\left[(abs)\ i\left(\frac{R^a - \rho^a}{R^a}\right)^m \left(\frac{\rho}{R}\right)^k\right] =$$

$$= \frac{1}{n\ R^{a\ m+k}}\Sigma[(abs)i\rho^k\ (R^a - \rho^a)^m].$$

This equation presents the most common form for U-G. The choice of the quantities a, k, m, R, as already mentioned, depends on the object of the determination of the U-G. For the U-G with respect to the surface areas of the drops, $k = 2$ is the logical choice. If, for reasons already stated, we put $a = 2$, $m = 2$, and $R = r_m$, we then obtain the U-G:

$$\Phi = \frac{1}{n\ r_m^6}\Sigma[i\ \rho^2\ (r_m^2 - \rho^2)^2].$$

For brevity this U-G is designated by Φ_I. We then have:

$$\Phi_I = \frac{1}{n}\left[\frac{1}{r_m^2}\Sigma(i\rho^2) - \frac{2}{r_m^4}\Sigma(i\rho^4) + \frac{1}{r_m^6}\Sigma(i\rho^6)\right] .$$

This expression represents the U-G of a mixture with respect to the combustion characteristics and determines the efficiency of an atomization together with the mean radius r_m of the drops. The smaller the values of r_m and Φ_I, just so much better is the atomization for an engine. But while the quantity r_m can be determined directly without knowing the size of the individual drops, the U-G Φ can be determined only when the sizes of all the drops in the mixture are known. Φ_I can not be determined from the quantities V, O, L, n, or from the resulting mean values of the size of the drops, due to the two therein occurring sums $\Sigma(i\rho^4)$ and $\Sigma(i\rho^6)$. The determination of the U-G Φ therefore involves the possibility of determining the size of all the drops and consumes much time.

We must therefore endeavor to discover a value for the U-G which can be deduced from the quantities V, O, L, n, and which can give a criterion for the U-G of a mixture, even though it may not fulfill all the requirements. Such a possibility presents itself if, instead of the U-G with respect to the surface areas of the drops, we determine the U-G with respect to the radii of the drops. Thereby we must put $a = 1$ and $k = 1$. Again, let $m = 2$ and $R = r_m$. From the general equation for Φ we then obtain the value

$$\Phi = \frac{1}{n\, r_m^{\,3}} \; \Sigma\left[i \; \rho \; (r_m - \rho)^2\right] .$$

For brevity let this value be represented by Φ_{II}. Then

$$\Phi_{II} = \frac{1}{n\, r_m^3}\left[\Sigma(i\,\rho\, r_m^2) - 2\,\Sigma(i\,\rho^2 r_m) + \Sigma(i\,\rho^3)\right]$$

$$= \frac{1}{n}\left[\frac{1}{r_m}\Sigma(i\rho) - \frac{2}{r_m^2}\Sigma(i\,\rho^2) + \frac{1}{r_m^3}\Sigma(i\,\rho^3)\right]$$

$$\Phi_{II} = \frac{(r_m)}{r_m} - 2\,\frac{(r_m)_1^2}{r_m^2} + \frac{(r_m)_2^3}{r_m^3}.$$

Thereby the U-G Φ_{II} is reduced to the determination of the mean values of the size of the drops. Care must be exercised, however, that the U-G Φ_{II} does not give the U-G with respect to the surface areas, but with respect to the radii of the drops. It deviates therefore more or less from U-G Φ_I and gives only an approximate criterion for the U-G with respect to the combustion characteristics of mixtures.

For the determination of Φ_{II}, the quantities V, O, L, n, or the mean values (r_m), $(r_m)_1$, $(r_m)_2$, and r_m must be known. For the direct determination of the quantities V, O, n, suitable methods are known, but the quantity L and the mean value (r_m) have thus far been impossible to determine without knowing the sizes of all the separate drops of the mixture. No practical application of the U-G Φ_{II} is therefore possible at the present time.

In addition to the U-G Φ_{II}, still other approximate values can be similarly derived, but I will refrain from their presentation, as I have found no satisfactory form.

Although no very practical importance attaches to the determination of the U-G, due to the present lack of suitable methods and to the troublesomeness of the process, it still seemed advisable to investigate, as thoroughly as possible, the definitive relations for the determination of the U-G, because, in determining the fineness of an atomization, it is often sought to obtain, in some simple way, at least an approximate idea of the uniformity, even when the size of the separate drops is not known. It is then necessary to estimate the course of the frequency curve and, from the area of the enclosed surface, to form an idea of the U-G of the mixture . (Compare a similar suggestion for estimating the U-G in an article by F. Häuser and G. Strobl, "Die Messung der Tropfengrösse bei zerstaubten Flussigkeiten," "Zeitschrift fur technische Physik," 5, 1924, p.157.) If the values of the U-G thus obtained are to be of practical importance, it is nevertheless necessary to find out whether and to what extent they agree with the U-G values obtained on the basis of the above-derived relation for Φ_I . It is conceivable that we can thus find an equation which will give, on the basis of the measurable quantities V, O, and n, a sufficiently accurate value for the U-G of an atomization.

In the preceding pages we have defined the terms "fineness" and "uniformity" of a mixture, which determine its efficiency. The latter is increased by decreasing the mean size r_m of the drops and the lack of uniformity Φ_I, which can be determined only when the size of the individual drops of the mixture is known, while the mean size of the drops can be determined directly, i.e., without determining the size of the individual drops.

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